Strongly Magnetized Sources
Alsacian X-Ray Polarimetry Meeting

Jeremy S. Heyl

14 November 2017

Ilaria Caiazzo, Roberto Mignami, Roberto Taverna, Roberto Turolla, Silvia Zane, and others.
Outline

QED Effective Action
  Birefringence

How It Works
  The Polarization-Limiting Radius
  Vacuum-Plasma Resonance

Sources
  Magnetars
  X-ray Pulsars

Summary
An Old Prediction

Folgerungen aus der Diracschen Theorie des Positrons.


Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwell'schen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:
An Old Prediction

The electrodynamics of the vacuum based on the quantum theory of the electron

V. WEISSKOPF


[Weisskopf's paper was originally printed with the following English-language abstract.]

This paper deals with the modifications introduced into the electrodynamics of the vacuum by Dirac’s theory of the positron. The
An Old Prediction

Folgerungen aus der Diracschen Theorie des Positrons.


The electrodynamics of the vacuum based on the quantum theory of the electron

V. Weisskopf

On Gauge Invariance and Vacuum Polarization

Julian Schwinger
Harvard University, Cambridge, Massachusetts
(Received December 22, 1950)

This paper is based on the elementary remark that the extraction of gauge invariant results from a formally gauge invariant theory is ensured if one employs methods of solution that involve only gauge covariant quantities. We illustrate this statement in connection with the problem of vacuum polarization by a prescribed electromagnetic field. The vacuum current of a charged Dirac field, which can be expressed in terms of the Green's function of that field, implies an addition to the action integral of the electron field, and a spin zero neutral meson arising from the polarization of the proton vacuum. We obtain approximate, gauge invariant expressions for the effective interaction between the meson and the electromagnetic field, in which the nuclear coupling may be scalar, pseudoscalar, or pseudovector in nature. The direct verification of equivalence between the pseudoscalar and pseudovector interactions only requires a proper statement of the limiting processes involved. For arbitrarily varying fields, perturbation methods can...
For a magnetic field the effective action is the free energy of the system (actually minus the free energy).

\[
\Gamma[A_\mu^0] = \int d^4x \left( -\frac{1}{4} F_{\mu\nu}^0 F^{0,\mu\nu} \right) - i\hbar \text{Tr} \ln \left[ \frac{\not{\nabla} - m}{\not{\phi} - m} \right]
\]
The QED Lagrangian

\[ \mathcal{L}_{\text{eff}} = \frac{\hbar}{8\pi^2} B_k^2 \int_0^\infty \frac{d\zeta}{\zeta} e^{-i\zeta} \left[ \frac{ab}{B_k^2} \coth \left( \frac{\zeta a}{B_k} \right) \cot \left( \frac{\zeta b}{B_k} \right) - \text{CT} \right] \]

where

\[ (b - ia)^2 = (B - iE)^2 = |B|^2 - |E|^2 - 2iE \cdot B \]

\[ [2(b - ia)^2] = F_{\mu\nu} F_{\mu\nu} + i\epsilon_{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \equiv I + iJ \]

and

\[ \text{CT} = \frac{1}{\zeta^2} + \frac{1}{3} \frac{a^2 - b^2}{B_k^2} (a^2 - b^2) \]

Heisenberg-Euler, Weisskopf, Schwinger
To understand the interaction of light with the magnetized vacuum, we imagine expanding the action for a uniform field plus a small photon field,

\[ E = E_0 + \delta E, \quad B = B_0 + \delta B, \quad F^{\mu \nu} = (F_0)^{\mu \nu} + f^{\mu \nu}. \]

We have two possibilities.

1. \( k \lambda_e \ll 1 \): we pretend that the photon field is also uniform and expand the effective Lagrangian density.
To understand the interaction of light with the magnetized vacuum, we imagine expanding the action for a uniform field plus a small photon field,

$$\mathbf{E} = \mathbf{E}_0 + \delta \mathbf{E}, \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}, F^{\mu\nu} = (F_0)^{\mu\nu} + f^{\mu\nu}.$$

We have two possibilities.

1. $k \lambda_e \ll 1$: we pretend that the photon field is also uniform and expand the effective Lagrangian density.
2. $k \lambda_e \gtrsim 1$: we have to expand the action itself.
\[ S = S_0 + \frac{1}{2} f_{\mu\nu} f_{\alpha\beta} \frac{\delta^2 S}{\delta f_{\mu\nu} \delta f_{\alpha\beta}} \]
\[ \Delta n = 4 \times 10^{-24} T^{-2} B^2 \]

What could be a signature of this birefringence?

- A time delay: \( \Delta t \sim 10^{-3} R/c \sim 10\text{ns} \)?
$$\Delta n = 4 \times 10^{-24} T^{-2} B^2$$

What could be a signature of this birefringence?

- A time delay: $\Delta t \sim 10^{-3} R/c \sim 10\text{ns}$?
- This seems a bit too subtle.
Liquid Crystal Displays
Liquid Crystal Displays

[Diagram of liquid crystal display]

Wikipedia

Jeremy S. Heyl  Strongly Magnetized Sources  Alsacian X-Ray Polarimetry
Kubo and Nagata (1983) present a concise way to characterize the evolution of the polarization of light through a medium; they simply write an equation to track the four Stokes parameters of the polarization light.

\[
\frac{\partial s}{\partial l} = \hat{\Omega} \times s
\]

where \( |\hat{\Omega}| = \Delta k \). The vector \( s = (S_1, S_2, S_3)/S_0 \) or \( (Q, U, V)/I \).
Stokes Parameters and the Poincaré Sphere

An important analytic solution.

What if $\frac{\partial \hat{\Omega}}{\partial l} = \hat{\Upsilon} \times \hat{\Omega}$?

1. Move into frame that corotates with $\hat{\Omega}$.

2. In this frame we have

$$\frac{\partial \mathbf{s}}{\partial l} = (\hat{\Omega} - \hat{\Upsilon}) \times \mathbf{s}$$

3. $\mathbf{s}$ orbits $\hat{\Omega}_{\text{Eff}}$ if

$$\left| \hat{\Omega} \left( \frac{1}{|\hat{\Omega}|} \frac{\partial |\hat{\Omega}|}{\partial l} \right)^{-1} \right| \gtrsim 0.5$$

Heyl, Shaviv 00
The radius at which the polarization stops following the birefringence is called the polarization-limiting radius. Beyond here the modes are coupled.
The radius at which the polarization stops following the birefringence is called the polarization-limiting radius. Beyond here the modes are coupled. The polarization-limiting radius for a dipole field is

\[ r_{pl} \equiv \left( \frac{\alpha \nu}{45 \ c} \right)^{1/5} \left( \frac{\mu}{B_{\text{QED}} \sin \beta} \right)^{2/5} \]

\[ \approx 1.9 \times 10^7 \left( \frac{\mu}{10^{30} \ G \ \text{cm}^3} \right)^{2/5} \left( \frac{E}{4 \ \text{keV}} \right)^{1/5} (\sin \beta)^{2/5} \ \text{cm}, \]
Why does this matter?

\[ \frac{r_{pl}}{R} = 0 \]
Heyl, Shaviv, Lloyd 03
Why does this matter?

\[ \frac{r_{pl}}{R} = 0 \]
Heyl, Shaviv, Lloyd 03

\[ \frac{r_{pl}}{R} = 1.9 \text{ (AM Her, AMSP)} \]
Why does this matter?

$$r_{pl}/R = 0$$

Heyl, Shaviv, Lloyd 03
Why does this matter?

\[ \frac{r_{pl}}{R} = 0 \]

Heyl, Shaviv, Lloyd 03

\[ \frac{r_{pl}}{R} = 12 \text{ (XRP)} \]
Why does this matter?

\[ \frac{r_{pl}}{R} = 0 \]
Heyl, Shaviv, Lloyd 03

\[ \frac{r_{pl}}{R} = 12 \text{ (XRP)} \]
Quasi-Tangential Region Wang, Lai 09

Strongly Magnetized Sources Alsacian X-Ray Polarimetry
Why does this matter?

\[ \frac{r_{pl}}{R} = 0 \]

Heyl, Shaviv, Lloyd 03
Why does this matter?

\[ \frac{r_{pl}}{R} = 0 \]

Heyl, Shaviv, Lloyd 03

\[ \frac{r_{pl}}{R} = 76 \text{ (Magnetar)} \]
Places to Look

<table>
<thead>
<tr>
<th>Source</th>
<th>Radius</th>
<th>Magnetic Field</th>
<th>$\mu_{30}$</th>
<th>$r_{pl}$ at 4 keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetar</td>
<td>$10^6$</td>
<td>$10^{15}$</td>
<td>$10^{33}$</td>
<td>$3.0 \times 10^8$</td>
</tr>
<tr>
<td>XRP</td>
<td>$10^6$</td>
<td>$10^{12}$</td>
<td>$10^{30}$</td>
<td>$1.9 \times 10^7$</td>
</tr>
<tr>
<td>ms XRP</td>
<td>$10^6$</td>
<td>$10^9$</td>
<td>$10^{27}$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>AM Her</td>
<td>$10^9$</td>
<td>$10^8$</td>
<td>$10^{35}$</td>
<td>$1.9 \times 10^9$</td>
</tr>
</tbody>
</table>
Deep in the atmosphere of the neutron star the plasma dominates, while outside the vacuum dominates. For large strengths of the magnetic field, the vacuum resonance may lie between the photospheres

\[ B \gtrsim B_l \approx 6.6 \times 10^{13} T_6^{-1/8} E_1^{-1/4} S^{-1/4} \text{G} \]

where \( S = 1 - e^{-E/kT} \).

This can have a strong effect on the appearance of spectral features and the high-energy slope. \cite{Ho, Lai 04}
Vacuum-Plasma Resonance

\[ B = 4 \times 10^{13} \text{ G} \]

\[ T_{\text{eff}} = 10^6 \text{ K} \]

\[ B = 10^{14} \text{ G} \]

\[ T_{\text{eff}} = 10^6 \text{ K} \]

\[ B = 4 \times 10^{13} \text{ G} \]

\[ B = 10^{14} \text{ G} \]

Ho, Lai 04

Jeremy S. Heyl

Strongly Magnetized Sources

Alsacian X-Ray Polarimetry Meeting
Vacuum-Plasma Resonance

The polarization-limiting radius is a critical parameter in understanding the behavior of charged particles in strong magnetic fields. This phenomenon is observed in various astrophysical sources, such as pulsars and active galactic nuclei. The graph illustrates how the polarization of emitted radiation changes with energy and magnetic field strength, highlighting the resonant behavior at certain energies.

Lai, Ho 03

Jeremy S. Heyl

Strongly Magnetized Sources

Alsacian X-Ray Polarimetry
Magnetar Emission

Caiazzo & Heyl 2016; 4U 0142+61

Taverna et al. 2016; SGR 1806-20 (350ks)
Realistic Hydrogen Atmosphere

Heyl, Shaviv, Lloyd 03
RX J1856.5-3754

Mignami et al. 2016
Her X-1

Caiazzo & Heyl 2017
Her X-1

Caiazzo & Heyl 2017
<table>
<thead>
<tr>
<th>Source</th>
<th>Radius</th>
<th>Magnetic Field</th>
<th>$\mu_{30}$</th>
<th>$r_{pl}$ at 4 keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetar</td>
<td>$10^6$</td>
<td>$10^{15}$</td>
<td>$10^{33}$</td>
<td>$3.0 \times 10^8$</td>
</tr>
<tr>
<td>XRP</td>
<td>$10^6$</td>
<td>$10^{12}$</td>
<td>$10^{30}$</td>
<td>$1.9 \times 10^7$</td>
</tr>
<tr>
<td>ms XRP</td>
<td>$10^6$</td>
<td>$10^9$</td>
<td>$10^{27}$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>AM Her</td>
<td>$10^9$</td>
<td>$10^8$</td>
<td>$10^{35}$</td>
<td>$1.9 \times 10^9$</td>
</tr>
<tr>
<td>Black Hole</td>
<td>$10^{6+}$</td>
<td>?</td>
<td></td>
<td>N/A</td>
</tr>
</tbody>
</table>